REVIEWS AND DESCRIPTIONS OF TABLES AND BOOKS

The numbers in brackets are assigned according to the American Mathematical Society classification scheme. The 1980 Mathematics Subject Classification (1985 Revision) can be found in the December index volumes of Mathematical Reviews.

1[65–01, 65M60, 65N30].—CLAES JOHNSON, *Numerical Solution of Partial Differential Equations by the Finite Element Method*, Cambridge Univ. Press, Cambridge, 1987, 278 pp., 23 cm. Price $69.50 hardcover, $24.95 paperback.

This text is an introduction to the finite element method and is classroom tested by its author in upperclass engineering courses at his home institution, Chalmers. The prerequisites are given as follows: "Basic courses in advanced calculus and linear algebra and preferably some acquaintance with the most well-known linear partial differential equations in mechanics and physics ...". Actually, at times, quite sophisticated tools are used, such as a priori estimates for elliptic problems, for the equation $\operatorname{div} v = q$, and for the behavior of downstream and parabolic layers in convection-dominated problems. The basic framework is that of bilinear forms on Hilbert spaces. Although the basic concepts are clearly explained and the more sophisticated ones referenced, I consider the present book as somewhat more demanding, mathematically, than, say, the excellent very elementary introduction to the subject by White [1]. Considering the situation in the US, by no means is the present text accessible only to the mathematics majors, but merely a sophomore study of calculus and linear algebra is insufficient preparation to really appreciate the presentation.

And appreciated it should be! In my opinion it is a masterful introduction to the subject. Its two main distinguishing features are: a thoroughly modern and up-to-date treatment, and the inclusion of some topics of current research interest. This sets it apart from most introductory texts, where developments within the last five years are seldom treated. (There is always a price tag: a revised edition of this work will probably be called for within a few years.) The writing is brisk and lively while still rigorous; it is a distinct pleasure to read. It will give the student a sound and clean conceptual framework for the methods, modern practical recipes for their implementation, and, to tease his imagination, a study of some methods which have not yet been canonized. Also, it is organized in such a way that an instructor who wishes to expand on certain topics beyond the presentation given, while cutting down on other parts, will have little problem in doing so.

I proceed to describe the contents and note in passing that the author does not aim at covering everything. For example, eigenvalue problems are not even mentioned.

The first part of the book, Chapters 1–7, covers standard material. The author provides the usual modern cleancut conceptual framework for the method in elliptic problems, giving plenty of examples, goes into the actual construction of some finite elements, gives some simple error estimates and describes direct and iterative methods for the solution of the resulting systems of linear equations. There is no deadwood, e.g., the chapter on iterative methods goes more or less directly (treating first general gradient methods, for later purposes) to the conjugate gradient method, with preconditioning, and then briefly treats multigrid methods (SSOR and ADI are not even mentioned). The short Section 4.6 on the use of error estimates for adaptive mesh refinement is noteworthy.

Chapter 8 is devoted to parabolic problems. Most of the material is traditional, but the discontinuous Galerkin method, and the use of error estimates for time-step control, are treated, again showing the up-to-date nature of this book.

Chapter 9 is thoroughly unusual for an introductory text. Here recent developments in applications of the finite element method to hyperbolic, and convection-dominated parabolic, problems are given. The streamline diffusion and discontinuous Galerkin methods are treated. It should be thought-provoking for students to come in contact with this recent research material, which has not yet made it into the canon of practical methods.

Chapters 10–12 then treat more standard material, boundary element methods, mixed methods, and curved elements and numerical integration, respectively.

A chapter on nonlinear problems such as obstacle problems, minimal surfaces, the incompressible Euler and Navier-Stokes equations, and Burgers' equation, concludes the book. The treatment in this chapter is very sketchy, often just giving the method. In my opinion this chapter is weak; however, twelve good chapters out of thirteen is a fine batting average.

It is traditional for a reviewer to show that he has read the book by picking quarrels with the author: The section on preconditioning, 7.4, is unnecessarily brief. As examples, only incomplete factorizations are mentioned, and since these methods do not lead to condition numbers bounded independently of the meshsize, they hardly do justice to the idea of preconditioning. A more convincing elementary example is that of a variable coefficient problem on a logically rectangular mesh, which may, after a piecewise mapping argument, be preconditioned by a Poisson problem on a square with a uniform mesh. The latter problem can be solved fast, e.g., by first applying the FFT in one direction and then tridiagonal solvers over each line in the other direction. The resulting effective condition numbers are bounded independently of the meshsize. I suggest expansion of this section in future editions (or, by the instructor), and also of the section on multigrid methods, which does little more than say that there exists a marvellous method called multigrid.

While error estimates are used to motivate adaptive procedures, it is not elucidated how useful they are for checking correctness of programs. A student may well wonder why one bothers to derive error estimates of the form $Ch^r|u|_s$, where C and u are not known, until it is pointed out to him that knowledge of r *alone* presents an invaluable debugging tool.

On p. 237, in connection with mixed methods for the Stokes problem, the author remarks that it is not clear how to solve the resulting equations iteratively in an efficient way. However, in the notation of (11.14), the pressure θ satisfies the equation $B^TA^{-1}B\theta = -B^TA^{-1}F$. The matrices $B^TA^{-1}B$ have condition numbers bounded independently of the mesh size, and thus, e.g., the conjugate gradient method will converge rapidly. Each step involves solving a standard Poisson problem. After that, one easily solves for the velocities. Thus, an efficient iterative method is easily found.

On p. 254, in connection with an obstacle problem, the choice $K_h = \{v \in V_h : v \geq \psi \text{ in } \Omega\}$ for the approximate constraint set is given. This choice is hard to implement. Imposing the inequality only at nodes, say, is easier to implement (but somewhat harder to analyze).

In conclusion, this is an impeccable introduction to the subject for an audience with some mathematical maturity beyond sophomore calculus and linear algebra. I predict that, for many purposes, it will replace the well-known book by Strang and Fix, [2], which is, very naturally, out of date in many respects.

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1. R. E. WHITE, *An Introduction to the Finite Element Method with Applications to Nonlinear Problems*, Wiley-Interscience, New York, 1985. (Review **1**, *Math. Comp.*, v. 50, 1988, pp. 343–345.)

2. G. STRANG & G. FIX, *An Analysis of the Finite Element Method*, Prentice-Hall, Englewood Cliffs, N.J., 1973. (Review **35**, *Math. Comp.*, v. 28, 1974, pp. 870–871.)

2[65N30, 65M60, 76D05, 76D07].—GRAHAM F. CAREY & J. TINSLEY ODEN, *Finite Elements: Fluid Mechanics*, The Texas Finite Element Series, Vol. VI, Prentice-Hall, Englewood Cliffs, N.J., 1986, x+323 pp., $23\frac{1}{2}$ cm. Price $38.95.

This is the sixth in the series devoted to Finite Element methods for the numerical solution of problems in Mechanics governed by partial differential equations. The first four volumes were concerned with the general exposition of the method while the fifth volume specifically concentrated on problems in Solid Mechanics. As the authors point out, Finite Elements were originally used to solve problems in Structural Mechanics, and their application to Fluid Dynamics is comparatively recent, a prerequisite for this being the formulation of basic problems in Fluid Dynamics in variational form.

The volume is self-contained since the authors include a brief but complete description of the general Finite Element method in the first chapter, giving a lucid explanation in terms of problems associated with Laplace's equation. The second chapter, dealing with compressible flow, is concerned mostly with transonic flow, and makes a strong case for applying Finite Element methods to such challenging problems as supercritical flow past airfoils, previously treated almost exclusively by finite difference methods. The technique for shock fitting is particularly appealing.

The third chapter is probably the most important in the volume, since it contains a thorough derivation of the Navier-Stokes equations, including the variational formulation of associated viscous flow problems. The latter leads to a detailed description of Finite Element methods, developed in turn for slow flows governed by the Stokes' equation, and for higher-speed steady flows governed by the full Navier-Stokes equations, including unsteady and compressibility effects. Applications to